ON THE RATE OF CONVERGENCE OF OPTIMAL SOLUTIONS OF MONTE CARLO APPROXIMATIONS OF STOCHASTIC PROGRAMS

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Abstract. In this paper we discuss Monte Carlo simulation based approximations of a stochastic programming problem. We show that if the corresponding random functions are convex piecewise linear and the distribution is discrete, then an optimal solution of the approximating problem provides an *exact* optimal solution of the true problem with probability one for sufficiently large sample size. Moreover, by using theory of Large Deviations, we show that the probability of such an event approaches one exponentially fast with increase of the sample size. In particular, this happens in the case of linear two (or multi) stage stochastic programming with recourse if the corresponding distributions are discrete. The obtained results suggest that, in such cases, Monte Carlo simulation based methods could be very efficient. We present some numerical examples to illustrate the involved ideas.

Key words. Two-stage stochastic programming with recourse, Monte Carlo simulation, Large Deviations theory, convex analysis

AMS subject classifications. 90C15, 90C25

1. Introduction. We discuss in this paper Monte Carlo approximations of stochastic programming problems of the form

$$\min_{x\in\Theta} \left\{ f(x) := \mathbb{E}_P h(x,\omega) \right\}, \tag{1.1}$$

where P is a probability measure on a sample space $(\Omega, \mathcal{F})$, Θ is a subset of $\mathbb{R}^m$ and $h : \mathbb{R}^m \times \Omega \to \mathbb{R}$ is a real valued function. We refer to the above problem as the "true" optimization problem. By generating an independent identically distributed (i.i.d.) random sample $\omega^1, ..., \omega^N$ in $(\Omega, \mathcal{F})$, according to the distribution P, one can construct the corresponding approximating program

$$\min_{x\in\Theta} \left\{ \hat{f}_N(x) := N^{-1} \sum_{j=1}^{N} h(x, \omega^j) \right\}. \tag{1.2}$$

An optimal solution $\hat{x}_N$ of (1.2) provides an approximation (an estimator) of an optimal solution of the true problem (1.1).

There are numerous publications where various aspects of convergence properties of $\hat{x}_N$ are discussed. Suppose that the true problem has a nonempty set A of optimal solutions. It is possible to show that, under mild regularity conditions, the distance $\operatorname{dist}(\hat{x}_N, A)$, from $\hat{x}_N$ to the set A, converges with probability one (w.p.1) to zero as $N \to \infty$. There is a vast literature in Statistics dealing with such consistency properties of empirical estimators. In the context of stochastic programming we can mention recent works [9],[14],[17], where this problem is approached from the point of view of the epiconvergence theory.

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